Bipartite Bell Inequalities for Hyperentangled States

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We show that bipartite Bell inequalities based on the Einstein-Podolsky-Rosen criterion for elements of reality and derived from the properties of some hyperentangled states allow feasible experimental verifications of the fact that quantum nonlocality grows exponentially with the size of the subsystems, and Bell loophole-free tests with currently available photodetection efficiencies.

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Bell's theorem states that quantum mechanics cannot be reproduced by any local realistic theory [1]. Therefore, either "there must be a mechanism whereby the setting of one measuring device can influence the reading of another instrument, however remote" [1], or we must give up on the idea that some physical observables possess definite values. This result is usually referred to as "quantum nonlocality." The violation of a Bell inequality (BI) is a standard method used to identify quantum nonlocality.

A BI is a constraint imposed by local realistic theories on the values of a linear combination $\hat{\beta}$ of the averages (or probabilities) of the results of experiments on two or more separated systems. It takes the form $\hat{\beta} \leq \beta$, where the bound β is the maximal possible value of $\hat{\beta}$ allowed by the local realistic theories. There are two types of BIs depending on how we define "local realistic theories."

CHSH-BIs.—The most common BIs belong to the Clauser-Horne-Shimony-Holt (CHSH) type [2, 3], in which local realistic theories are defined as those in which: (i) the probabilities of the outcomes of *all* local observables are predetermined, and (ii) these probabilities cannot be affected by spacelike separated measurements. For two separated systems 1 and 2, in any CHSH-BI, $\hat{\beta}$ takes the following general form

$$\hat{\beta} = \sum_{i=1}^{m} \sum_{j=1}^{n} c(i,j) \langle A_1^{(i)} B_2^{(j)} \rangle, \tag{1}$$

where $c(i,j)$ are certain constant coefficients, i and j are indices (discrete or continuous) distinguishing the possible experiments on system 1 and 2, respectively, and $\langle A_1^{(i)} B_2^{(j)} \rangle$ is a correlation function (the average of the product of the observables measured on 1 and 2). If $A_1^{(i)}$ and $B_2^{(j)}$ are spacelike separated experiments, from assumptions (i) and (ii) follow that the correlation must take the form

$$\langle A_1^{(i)} B_2^{(j)} \rangle = \sum_{\lambda} f_1(A_1^{(i)}, \lambda) g_2(B_2^{(j)}, \lambda) p(\lambda), \tag{2}$$

where $f_1(A_1^{(i)}, \lambda)$ $[g_2(B_2^{(j)}, \lambda)]$ is a function which gives the value of the experiment $A_1^{(i)}$ $(B_2^{(j)})$ on subsystem 1 (2), and λ is a summation (or integration) parameter which allows the description to have a probabilistic nature; $p(\lambda)$ is the distribution of the parameter.

Curiously, the original BI [1] does *not* belong to this type. Any product state satisfies any CHSH-BI. Therefore, separable states (which are convex combinations of product states) satisfy them too. However, some separable states violate the original BI [4]. The explanation is that the original BI is based on assumptions that are not satisfied by these separable states [5].

EPR-BIs.—The premise of the original BI is the Einstein-Podolsky-Rosen (EPR) criterion for the existence of elements of reality: "if, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity" [6]. The EPR criterion establishes *two* conditions for the existence of elements of reality. Firstly, *perfect predictability*: it must be possible to predict them with certainty. Secondly, *locality*: the prediction must be based on a measurement that exerts no disturbing influence upon them.

Bell's original inequality is based on an *equality*: it is based on the fact that, for the two-qubit singlet state, the results of measuring the same observable B on both qubits are perfectly anticorrelated, $\langle BB \rangle = -1$. Bell uses this equality in two ways: to guarantee that all local observables are EPR elements of reality, and in the derivation of the inequality.

What makes EPR-BIs so attractive is that the EPR criterion seems almost unavoidable: EPR do *not* assume that all local experiments should have predefined values; the existence of predefined values is assumed to explain why they can be predicted from remote measurements. On the other hand, the advantage of the CHSH-BIs is that they do not depend on the properties of a particular state. The problem for testing EPR-BIs is the difficulty of having perfect correlations in actual experiments.

The work by Greenberger, Horne, and Zeilinger (GHZ) [7], and Mermin [8] stimulated a renewed interest in the local realistic theories of the EPR type. Moreover, the development of quantum technologies in the last decade has opened new possibilities.

Almost perfect predictability.—The first interesting development is that we can prepare two-particle states with

almost perfect correlations. This almost perfect remote predictability opens the door for the testing of the EPR-BIs. There are two possible strategies: One is to relax the EPR criterion and define elements of reality as those that can be predicted with *almost* perfect certainty [9]. This definition would automatically extend the validity of the BI to all prepared pairs. The second strategy is valid if we can prepare pairs so that a very high fraction of them allows perfect predictability. Then we can assume the original EPR criterion. In this case, the EPR-BI is legitimate only for a fraction of pairs. However, we still can obtain conclusive experimental results by performing tests on all the pairs, and then calculating how the fraction of the pairs for which the inequality is not valid can affect these experimental results.

Hyperentanglement and detection efficiency.—The second interesting development is the possibility of preparing pairs of particles in hyperentangled states, i.e. entangled in several degrees of freedom [10]. Hyperentanglement has been demonstrated in recent experiments with two photons entangled in two degrees of freedom (polarization and path) [11], and in three degrees of freedom (polarization, path, and time-energy) [12]. Indeed, time-bin entanglement would allow us to encapsulate a higher number of qubits [13]. This is very interesting for the following reason: imagine we have $2N$ qubits distributed in $2N$ particles; then, to "reveal" $2N$ EPR elements of reality we would need to activate $2N$ single-particle detectors, something that occurs with probability η^{2N}, being η the efficiency of each of the single-particle detectors. However, if we have $2N$ qubits encapsulated in 2 particles; then, to reveal $2N$ EPR elements of reality we would only need to activate 2 single-particle detectors, something that occurs with probability η^2. The interest of this is related to the fact that the main obstacle for a loophole-free test of BIs is that η is very low for photons.

Simultaneous EPR elements of reality.—A third motivation for exploring EPR-BIs is related with the experimental capability of entangling higher dimensional subsystems. While the $\hat{\beta}$ corresponding to bipartite CHSH-BIs takes the same form (1) irrespective of the dimension d of the Hilbert space describing the local subsystems, the $\hat{\beta}$ corresponding to bipartite EPR-BIs can take different forms depending on d. The interesting point is that if $d > 2$, there are *compatible* local observables which can be regarded as *simultaneous* EPR elements of reality [14]. For instance, suppose that $A_1^{(1)}$ and $A_1^{(2)}$ are observables on particle 1 represented by *commuting* operators. Now there is a new possibility: it can so happen that there are a quantum state $|\phi\rangle$ and two local observables $B_2^{(1)}$ and $B_2^{(2)}$, on particle 2 so that $\langle\phi|A_1^{(1)}B_2^{(1)}|\phi\rangle = 1$ and $\langle\phi|A_1^{(2)}B_2^{(2)}|\phi\rangle = 1$. Therefore, since $A_1^{(1)}$ and $A_1^{(2)}$ can be remotely predicted with certainty, then *both* $A_1^{(1)}$ and $A_1^{(2)}$ are EPR elements of reality *simultaneously.* In principle, it could happen that, since $A_1^{(1)}$ and $A_1^{(2)}$ are measured on the same subsystem, the measurement of one of them may disturb the value of the other. The remarkable point is that this hypothetical disturbance can be discarded if a spacelike separated observer *can* predict with certainty the values of $A_1^{(1)}$ or $A_1^{(2)}$, not only when they are measured separately, but also when they are measured together.

Therefore, for higher dimensional bipartite EPR-BIs, $\hat{\beta}$ can contain terms like $\langle A_1^{(1)}A_1^{(2)}B_2^{(1)}\rangle$, and the general form of $\hat{\beta}$ is

$$\begin{aligned} \hat{\beta} \;=\; & \sum_{i=1}^{m}\cdots\sum_{j=1}^{n}\sum_{k=1}^{p}\cdots\sum_{l=1}^{q} c(i,\ldots,j,k,\ldots,l) \\ & \times\langle A_1^{(i)}\ldots A_1^{(j)}B_2^{(k)}\ldots B_2^{(l)}\rangle, \end{aligned} \tag{3}$$

where all of the local observables are EPR elements of reality (for certain states), and all the local observables appearing in the same average are compatible. Examples of bipartite EPR-BIs have been introduced [15, 16, 17] and experimentally tested [11] recently.

The aim of this Letter is to explore the merits of these new equalities-based BIs when we move to higher dimensions. For this purpose, we derive a bipartite higher dimensional EPR-BI based on the properties of a hyperentangled state, and show that it can be used to solve two still-open experimental problems in quantum mechanics.

Growing with size nonlocality.—There was a time when it was thought that quantum nonlocality would decrease as the size of the system grows, as a manifestation of some intrinsic aspect of the transition from quantum to classical behavior. By "size" we mean either the number of particles or the number of internal degrees of freedom. However, Mermin [18] showed that the correlations found by n spacelike separated observers that share n qubits in a GHZ state violate a n-party (with $n \geq 3$) BI by a factor that increases *exponentially* with n. An experimental verification of this exponentially growing nonlocality using GHZ states is difficult because it requires n spacelike separated measurements, and because n-party GHZ states' sensitivity to decoherence also grows with n [19].

The ratio $\beta_{\text{EXP}}/\beta_{\text{EPR}}$, where β_{EXP} is the experimental value of $\hat{\beta}$ (which is supposedly similar to β_{QM}), and β_{EPR} is the maximal possible value of $\hat{\beta}$ allowed by the local realistic theories of the EPR-type, is a good measure of nonlocality, since it is related both to the number of bits needed to communicate nonlocally in order to emulate the experimental results by a local realistic theory, and also to the minimum detection efficiency needed for a loophole-free experiment (as explained below). In all known bipartite CHSH-BIs this ratio is almost *constant* with the number of internal levels of the local subsystems [13, 20]. However, this is not the case in the following EPR-BI.

Consider two particles 1 and 2 prepared in the state

$$|\Psi\rangle = \bigotimes_{j=1}^{N} |\psi\rangle^{(j)}, \tag{4}$$

where

$$\begin{aligned}|\psi\rangle^{(j)} &= \frac{1}{2}\Big(|00\rangle_1^{(j)}|00\rangle_2^{(j)} + |01\rangle_1^{(j)}|01\rangle_2^{(j)} + |10\rangle_1^{(j)}|10\rangle_2^{(j)} \\ &\quad -|11\rangle_1^{(j)}|11\rangle_2^{(j)}\Big). \end{aligned}\tag{5}$$

The state $|\Psi\rangle$ encapsulates $4N$ qubits in two particles. Consider the following single qubit observables:

$$X_k^{(j)} = \sigma_x^{(j)} \otimes \mathbb{1}^{(j)},\, Y_k^{(j)} = \sigma_y^{(j)} \otimes \mathbb{1}^{(j)},\ Z_k^{(j)} = \sigma_z^{(j)} \otimes \mathbb{1}^{(j)}, \tag{6}$$

$$x_1^{(j)} = \mathbb{1}^{(j)} \otimes \sigma_x^{(j)},\, y_2^{(j)} = \mathbb{1}^{(j)} \otimes \sigma_y^{(j)},\ z_2^{(j)} = \mathbb{1}^{(j)} \otimes \sigma_z^{(j)}, \tag{7}$$

where k denotes particle k, σ_x is the Pauli matrix in the x direction, and $\mathbb{1}$ is the identity matrix in a two-dimensional Hilbert space. For the state $|\Psi\rangle$, each and every one of these $7N$ single qubit observables $X_1^{(j)}$, $Y_1^{(j)}$, $x_1^{(j)}$, $X_2^{(j)}$, $Y_2^{(j)}$, $y_2^{(j)}$, and $z_2^{(j)}$ can be regarded as an EPR element of reality, since it satisfies the following $7N$ equalities representing perfect correlations:

$$\langle X_1^{(j)} X_2^{(j)} z_2^{(j)}\rangle = 1, \quad \langle Y_1^{(j)} Y_2^{(j)} z_2^{(j)}\rangle = -1, \tag{8}$$

$$\langle x_1^{(j)} Z_2^{(j)} x_2^{(j)}\rangle = 1, \tag{9}$$

$$\langle X_1^{(j)} z_1^{(j)} X_2^{(j)}\rangle = 1, \quad \langle Y_1^{(j)} z_1^{(j)} Y_2^{(j)}\rangle = -1, \tag{10}$$

$$\langle Z_1^{(j)} y_1^{(j)} y_2^{(j)}\rangle = -1, \quad \langle z_1^{(j)} z_2^{(j)}\rangle = 1. \tag{11}$$

Therefore, we can define

$$\begin{aligned}\beta &= \langle X_1^{(1)}X_2^{(1)}z_2^{(1)}\ldots X_1^{(N-1)}X_2^{(N-1)}z_2^{(N-1)}X_1^{(N)}X_2^{(N)}z_2^{(N)}\rangle \\ &\quad -\langle X_1^{(1)}X_2^{(1)}z_2^{(1)}\ldots X_1^{(N-1)}X_2^{(N-1)}z_2^{(N-1)}Y_1^{(N)}Y_2^{(N)}z_2^{(N)}\rangle \\ &\quad +\langle X_1^{(1)}X_2^{(1)}z_2^{(1)}\ldots X_1^{(N-1)}X_2^{(N-1)}z_2^{(N-1)}X_1^{(N)}x_1^{(N)}Y_2^{(N)}y_2^{(N)}\rangle \\ &\quad +\langle X_1^{(1)}X_2^{(1)}z_2^{(1)}\ldots X_1^{(N-1)}X_2^{(N-1)}z_2^{(N-1)}Y_1^{(N)}x_1^{(N)}X_2^{(N)}y_2^{(N)}\rangle \\ &\quad -\langle X_1^{(1)}X_2^{(1)}z_2^{(1)}\ldots Y_1^{(N-1)}Y_2^{(N-1)}z_2^{(N-1)}X_1^{(N)}X_2^{(N)}z_2^{(N)}\rangle + \ldots \\ &\quad +\langle Y_1^{(1)}x_1^{(1)}X_2^{(1)}y_2^{(1)}\ldots Y_1^{(N)}x_1^{(N)}X_2^{(N)}y_2^{(N)}\rangle,\end{aligned}\tag{12}$$

which contains 4^N expectation values. For measuring, for instance, $X_1^{(1)}x_1^{(1)}\ldots X_1^{(N)}x_1^{(N)}$ on particle 1, we use an analyzer that separates the two possibilities of each of the $2N$ qubit observables $X_1^{(1)}$, $x_1^{(1)}$, …, $X_1^{(N)}$, $x_1^{(N)}$. This analyzer is backed up by 4^N particle detectors, one for each of the possible outcomes. Therefore, each particle detection gives the value of $2N$ observables. Each observer can choose between the 4^N local experiments. The choice of experiment and the detection of particle 1 are assumed to be random and spacelike separated from those of particle 2.

As can be easily checked, in any EPR-type local realistic theory, $\beta_{\rm EPR} = 2^N$, while the value predicted by quantum mechanics is $\beta_{\rm QM} = 4^N$, which violates the EPR bound by an amount which grows as $\beta_{\rm QM}/\beta_{\rm EPR} = 2^N$, assuming perfect states and measurements. The remarkable point is that this "exponentially growing with size nonlocality" can be demonstrated by actual experiments if we use two-particle hyperentangled states. In practice, we do not have perfect correlations but

$$\langle X_1^{(1)}x_1^{(1)}\ldots X_1^{(N)}x_1^{(N)}Y_2^{(1)}y_2^{(1)}\ldots Y_2^{(N)}y_2^{(N)}\rangle = 1-\epsilon, \tag{13}$$

where $\epsilon \approx 0.15$ [21]. In a worst-case scenario, each of the terms in $\hat{\beta}$ is affected by a similar error. Since the number of terms in $\hat{\beta}$ is 4^N, then we should take into account that our value for $\beta_{\rm EPR}$ could be increased to

$$\beta'_{\rm EPR} \approx 2^N + 4^N 0.15. \tag{14}$$

Also, we must take into account the imperfection in the preparation of the state which, in practice, is not $|\Psi\rangle$, but $\rho = p|\Psi\rangle\langle\Psi| + (1-p)\rho'$, with $p \approx 0.98$, and the specific form of the term ρ' depends on the physical procedure used to prepare and distribute the state. Therefore, the expected experimental value of $\hat{\beta}$ is

$$\beta'_{\rm QM} \approx 0.98 \times 4^N + 0.02, \tag{15}$$

The interesting point is that $\beta'_{\rm QM}$ still provides a significant violation of the inequality $\hat{\beta} \le \beta'_{\rm EPR}$, violation which exhibits a growing with size nonlocality. An experiment for observing this effect for lower values of N is feasible using currently available capabilities [21].

Loophole-free Bell experiments.—Experiments to test CHSH-BIs have fallen within quantum mechanics and, under certain additional assumptions, seem to exclude local realistic theories [22]. A particularly relevant loophole is the so-called detection loophole [23]. It arises from the fact that, in most experiments, only a small subset of all the created pairs are actually detected, so we need to assume that the detected pairs are a fair sample of the created pairs. Otherwise, it is possible to build a local model reproducing the experimental results. Closing the detection loophole using the CHSH inequality requires $\eta \geq 0.83$ [3, 24]. The best of currently available photodetection efficiency is $\eta = 0.33$ [25]. There are several proposals for loophole-free experiments [26]. Garg and Mermin suggested that "It is possible that $n \times n$ experiments with n larger than 2 can refute local realism with lower detector efficiencies" [24]. Nevertheless, this conjecture has not been proven to be true with known bipartite CHSH-BIs. However, this effect can be observed by using EPR-BIs.

Consider the previous EPR-BI for the state $|\Psi\rangle$. Let us calculate the minimum detection efficiency required for a loophole-free test. If $\mathcal{N}(\mathcal{AB}=1)$ is the number of pairs in which the product of the results of measuring, for instance, $\mathcal{A} = X_1^{(1)} x_1^{(1)} \ldots X_1^{(N)} x_1^{(N)}$ on particle 1 and $\mathcal{B} = Y_2^{(1)} y_2^{(1)} \ldots Y_2^{(N)} y_2^{(N)}$ on particle 2 is 1, and $\mathcal{N}$ is the total number of emitted pairs, then the corresponding correlation is $\langle \mathcal{AB} \rangle = [\mathcal{N}(\mathcal{AB}=1) - \mathcal{N}(\mathcal{AB}=-1)]/\mathcal{N}$. If η is the detection efficiency of each and every one of the 4^N particle detectors behind each analyzer, then the number of *detected* pairs in which the product of the results of measuring $\mathcal{A}$ on particle 1 and $\mathcal{B}$ on particle 2 is ± 1, is related with the theoretical number by

$$\mathcal{N}_{\mathrm{EXP}}(\mathcal{AB} = \pm 1) = \eta^2 \mathcal{N}(\mathcal{AB} = \pm 1). \tag{16}$$

On the other hand,

$$\begin{aligned} \mathcal{N} \; = \; & \mathcal{N}_{\mathrm{EXP}}(\mathcal{AB}=1) + \mathcal{N}_{\mathrm{EXP}}(\mathcal{AB}=-1) \\ & + \mathcal{N}_{\mathrm{EXP}}(\mathcal{A}=\pm 1, \mathcal{B}=0) + \mathcal{N}_{\mathrm{EXP}}(\mathcal{A}=0, \mathcal{B}=\pm 1) \\ & + \mathcal{N}_{\mathrm{EXP}}(\mathcal{A}=0, \mathcal{B}=0), \end{aligned} \tag{17}$$

where $\mathcal{N}_{\mathrm{EXP}}(\mathcal{A} = \pm 1, \mathcal{B} = 0)$ is the number of pairs in which when $\mathcal{A}$ is measured on particle 1 and $\mathcal{B}$ is measured on particle 2, and one detector corresponding to particle 1 is activated, but no detector corresponding to particle 2 is activated. We usually do not know $\mathcal{N}_{\mathrm{EXP}}(\mathcal{A}=0, \mathcal{B}=0)$, because we cannot know $\mathcal{N}$; however, the relation between them is

$$\mathcal{N}_{\mathrm{EXP}}(\mathcal{A}=0, \mathcal{B}=0) = (1-\eta)^2 \mathcal{N}. \tag{18}$$

The probability that two or more detectors corresponding to the same particle are activated simultaneously is assumed to be negligible. What we obtain in an experiment is

$$\begin{aligned} \langle \mathcal{AB} \rangle_{\mathrm{EXP}} \; = \; & [\mathcal{N}_{\mathrm{EXP}}(\mathcal{AB}=1) - \mathcal{N}_{\mathrm{EXP}}(\mathcal{AB}=-1)] \\ & \times [\mathcal{N} - \mathcal{N}_{\mathrm{EXP}}(\mathcal{A}=0, \mathcal{B}=0)]^{-1}. \end{aligned} \tag{19}$$

Therefore, substituting (16) and (18) in (19), we obtain

$$\langle \mathcal{AB} \rangle = \frac{\eta^2}{1-(1-\eta)^2} \langle \mathcal{AB} \rangle_{\mathrm{EXP}}. \tag{20}$$

Therefore, taking into account the detection efficiencies, the EPR-BI becomes

$$\frac{\eta^2}{1-(1-\eta)^2} \beta_{\mathrm{QM}} \leq \beta_{\mathrm{EPR}}, \tag{21}$$

where β_{QM} and β_{EPR} must be replaced by (15) and (14), if we take the errors in the state and the measurements into account. The remarkable point is that the minimum η required for a loophole-free test is a function of the ratio $\beta_{\mathrm{QM}}/\beta_{\mathrm{EPR}}$. For instance, assuming perfect states, for $N = 1$ we recover the value $\eta \approx 0.83$ [3, 24]. More interestingly, if we ask which is the value of N in order to get a loophole-free test assuming the best currently available efficiency $\eta = 0.33$ [25], the answer turns out to be $N \geq 6$ (i.e., 12 qubits per photon) taking into account the errors in the state and the measurements. Alternatively, we can use higher dimensional two-photon entangled states produced by fiber interferometers. This has the advantage that each photon can be sent through a different fiber and thus the local measurements can be spacelike separated. The difficulty of having lower photodetection efficiencies at telecom wavelengths could be compensated by the possibility of producing entangled states in arbitrary high dimension [13]. Therefore, this approach could close the detection loophole using currently available photodetection efficiencies.

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